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AERODYNAMIC AND ELECTROMAGNETIC PROPERTIES OF PLASMAS.(U)

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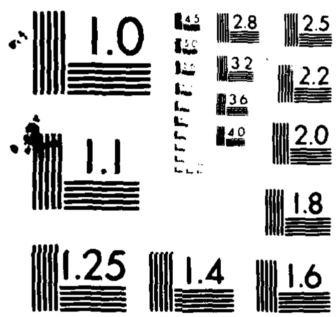

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AERODYNAMIC AND ELECTROMAGNETIC PROPERTIES OF PLASMAS

Final Report on the Research Project

AFOSR- 77-3211

The University of Michigan
Ann Arbor, Michigan 48109

October 1981

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I. Period Covered

December 1, 1976 - August 31, 1981

II. Personnel

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IV. Ph.D. Thesis

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Michael E. Grost

David Huff

James Turner

J. P. McKenna

Shui-Hung Hou

Michael W. Smiley

Robert A. Gardner

David Wagner

III. LIST OF REPORTS ISSUED IN THE PERIOD

December 1, 1976 - August 1, 1981

1. L. Cesari, A new method of successive approximations for nonselfadjoint nonlinear boundary value problems. To appear in the Acta of the Conference on Differential Equations, University of Uppsala, April 1977.

2. J. A. Smoller, and C. Bardos, Instabilité des solutions stationnaires pour des systemes de reaction diffusion. C. R. Acad. Sci. Paris, 285, ser. A, 249-252.

3. J. A. Smoller and C. Conley, Remarks on traveling wave solutions of nonlinear diffusion equations. To appear in Springer-Verlag Lecture Notes in Math., Vol. 525.

4. J. A. Smoller, E. Conway, and D. Hoff, Large time behavior of solutions of systems of nonlinear reaction-diffusion equations. To appear in SIAM J. Appl. Math.

5. J. A. Smoller and E. Conway, Diffusion and the classical ecological interactions. Asymptotics. To appear in the Acta of the Conference on Reaction Diffusion Equations.

6. M. E. Grost, An existence theorem for boundary value problems for nonlinear hyperbolic partial differential equations.

7. D. Ku, Boundary value problems and numerical estimates.

8. L. Cesari and M. B. Suryanarayana, An existence theorem for Pareto problems. To appear in Journal for Nonlinear Analysis.

9. L. Cesari and J. P. McKenna, Alternative problems and Grothendieck approximation properties.

10. L. Cesari, The duplication of frequency of laser radiation through nonlinear media.

11. E. D. Conway and J. A. Smoller, A comparison technique for systems of reaction diffusion equations.

12. L. Cesari, Remarks on the principle of uniform contraction.

13. L. Cesari, Boundary value problems for ordinary differential systems with discontinuous nonlinearities.

14. L. Cesari and R. Kannan, Existence of solutions of nonlinear hyperbolic equations.

15. L. Cesari and R. Kannan, Solutions of nonlinear hyperbolic equations at resonance.

16. L. Cesari and M. B. Suryanarayana, Upper semicontinuity properties of set valued function.

17. M. B. Suryanarayana, On Young's lemma for the Dirichlet integral.

18. J. Smoller, A. Tromba and A. Wasserman, Nondegenerate solutions of boundary value problems.

19. L. Cesari and V. E. Bononcini, Periodic solutions of ordinary differential equations.

20. S. H. Hou, Existence theorems for optimal control problems in Banach spaces.

21. L. Cesari, S. H. Hou and J. Turner, The duplication of frequency of laser radiation through nonlinear media. First and Second Part.

22. L. Cesari and D. Ku, A method of successive approximations for nonselfadjoint nonlinear boundary value problems.

23. C. Bardos, H. Matano and J. Smoller, Some results on the instability of solutions of reaction diffusion equations.

24. C. Conley and J. Smoller, Isolated invariant sets of parametrized systems of differential equations.

25. L. Cesari, The duplication of frequency in laser radiation through nonlinear media.

26. P. J. McKenna and Howard Shaw, The structure of the solution set of some nonlinear problems.

27. L. Cesari and M. B. Suryanarayana, On recent existence theorems in the theory of optimization.

28. L. Cesari, Existence theorems for nonlinear problems and numerical methods. Lectures given at the University of Bari, Italy.

29. J. A. Smoller and T. P. Liu, On the vacuum state for the isentropic gas dynamic equations.

30. J. A. Smoller and Arthur Wasserman, Global bifurcation of steady state solutions.

31. J. A. Smoller and C. Conley, Topological techniques in reaction-diffusion equations.

32. L. Cesari and H. W. Engl, Existence and uniqueness of solutions for nonlinear alternative problems in a Banach space.

33. R. F. Baum, An approximation procedure for solutions to Meyer problems with cost functional not of class C^1.

34. L. Cesari and R. Kannan, Periodic solutions of nonlinear wave equations.

35. M. B. Suryanarayana, Monotonicity and upper semicontinuity of multi-functions.

36. J. A. Smoller and C. Conley, Remarks on the stability of steady state solutions of reaction-diffusion equations.

37. L. Cesari and R. Kannan, Periodic solutions of nonlinear wave equations. In process of publication in Archive Rat. Mech. Anal.

38. L. Cesari and P. Bassanini, La duplicazione di frequenza nella radiazione laser. To appear in Rend. Accad. Lincei, Roma.

39. L. Cesari and R. Kannan, On the periodic solutions of a nonlinear differential equation. To appear in Proc. Amer. Math. Soc.

40. L. Cesari and R. Kanna, Periodic solutions of nonlinear wave equations with damping. To appear in Rend. Circolo Mat. Palermo.

41. L. Cesari and R. Kannan, Solutions of nonlinear wave equations with nonlinear damping. To appear in Journal for Partial Differential Equations.

42. L. Cesari and R. Kannan, An existence theorem for periodic solutions of nonlinear parabolic equations. To appear in Rend. Accad. Scienze Milano.

43. L. Cesari, Existence of solutions of hyperbolic problems. International Conference on nonlinear oscillations, Arlington, Texas, June 1980.

44. L. Cesari, Nonlinear boundary value problems for hyperbolic systems. International Conference on Dynamical Systems. Gainesville, Florida, February 27 - 29, 1981. To appear in a volume published by Academic Press.

45. Patrizia Pucci, Integrali di Riemann e di Burkill-Cesari, to appear in Rend. Mat. Univ. Roma.

46. Michael W. Smiley, A weak formulation of the 2-point boundary value problem for hyperbolic equations. To appear in Rend. Mat. Univ. Parma.

47. Michael W. Smiley, Existence theorems for linear hyperbolic boundary value problems at resonance. To appear in Annali Matem. Pura Appl.

48. Michael W. Smiley, Existence theorems for nonlinear hyperbolic boundary value problems at resonance. To appear in Nonlinear Analysis.

49. Shui-Hung Hou, Implicit function theorem on topological spaces. To appear in Applicable Mathematics.

50. Shui-Hung Hou, On property (Q) and other semicontinuity properties of multifunctions. To appear in Pacific Math. Journal.

51. Shui-Hung Hou, Controllability and feed-back systems. Submitted to Trans. Amer. Math. Soc.

52. Shui-Hung Hou, Existence theorems of optimal control problems in Banach spaces. To appear in Nonlinear Analysis.

V. *Summaries of recent reports*

Summaries prepared by L. Cesari

Report No. 37: L. Cesari and R. Kannan, Periodic solutions of nonlinear wave equations.

We consider the question of the existence of doubly periodic solutions of the nonlinear partial differential equations

$$(1)\quad \begin{aligned} &u_{tt}-u_{ss} = f(t,s,u), \\ &u(t+T,s) = u(t,s) = u(t,s+T) , \end{aligned}$$

and

$$(2)\quad \begin{aligned} &u_{ts} = f(t,s,u), \\ &u(t+T,s) = u(t,s) = u(t,s+T) , \end{aligned}$$

for a given T and where $f(t,s,u)$ is a given doubly period function of t,s,

$$f(t+T,s,u) = f(t,s,u) = f(t,s+T,u) .$$

The differential equations (1) and (2) are related since the transformation $t=x+y$, $s=x-y$ transforms one into the other. However, the boundary conditions do not match.

In this report we discuss the problem (2) for the case where the second member is of the form $f = \phi(t,s)+g(u)$, and $g(u)$ is a bounded function.

For the case in which g is globally monotone, and the case in which $f=\varepsilon f_o$ with ε a small parameter, results are already known (Vejvoda, Brezis, Nirenberg). Here we assume that g is not globally monotone, and there is no small parameter in the equations.

By an analysis based on functional analysis, the problem is reduced to a system of auxiliary and bifurcation equations, for which the existence of solutions is proved by Schauder's fixed point theorem.

In a joint paper with Patrizia Pucci (in preparation), the results will be transferred to problem (1).

Report No. 38: L. Cesari and P. Bassanini, La duplicazione di frequenza nella radiazione laser.

If a monochromatic laser beam is focused on a thin crystal,then after emerging the light shows a measurable component of double frequency. Thus, red laser light 6940 Å wave length after emerging from a thin quartz crystal has an ultraviolet component 3470 Å wave length. This phenomenon was discovered in 1960 and the physicists account rather well for this nonlinear phenomenon in physical term. If $0<x<a$ is the crystal, Graffi proposed as a rigorous mathematical model for the plane waves, the nonlinear Maxwell equations

$$-\partial H/\partial x = \varepsilon_2 \partial E/\partial t + \eta E \partial E/\partial t, \quad -\partial E/\partial x = \mu_o \partial H/\partial t \quad ,$$

$$0<x<a, \quad -\infty<t<+\infty \; .$$

Cesari showed that the natural boundary conditions for this model are

$$k\,E(0,t)+H(0,t) = w(t), \quad k\,E(a,t) - H(a,t) = 0 \;, \quad -\infty<t<+\infty \;,$$

where ε_2, η, μ, k are physical constants, $k>1$, and where $w(t)$ represents the incoming wave.

In previous work Cesari had proved existence and uniqueness theorems for a large class of hyperbolic systems with boundary conditions given by linear combinations of the unknowns on parallel hyperplanes (1975). This work had been motivated by this physical problem which is reducible to such class by algebraic manipulation. Also, a method of successive approximations had been derived for the numerical handling of such class of problems. Later Cesari with students in Ann Arbor, and Bassanini and students in Perugia (Italy) determined numerical solutions of the laser problem in typical situations. The numerical results agree with those expected and are in accord with the experimental data.

This Report is the official communication of the results to the Accademia Nazionale dei Lincei in Rome.

Report No. 39: L. Cesari and R. Kannan, On the periodic solutions of a nonlinear differential equation.

In connection with recent work by M. Scheckter on the ordinary differential equation $u'' + u + g(u) = h, u(0) = u(\pi) = 0$, g continuous and monotone in $(-\infty, +\infty)$, $h \in L_2[0,\pi]$, the authors prove that there is at least one solution provided $\limsup_{s\to+\infty} \gamma(s)/s = \gamma$, $|g(s)| \leq C + \gamma(s)$, C, γ constants, $\gamma<0.375$. This result improves the previous one of Scheckter who had obtained existence only for $|\gamma| \leq 0.05$. Note that in this problem a necessary upper bound for γ is 3.

Report No. 40: L. Cesari and R. Kannan, Periodic solutions of nonlinear wave equations with damping.

We consider the problem of existence of periodic solutions of period 2π in t of the following nonlinear wave equation with damping:

$$u_{tt}-u_{ss}+\beta u_t + g(u) = h(t,s),$$

$$u(t,0) = u(t,\pi) = 0\ ,\quad u(t+2\pi,s) = u(t,s),$$

$$0<s<\pi,\qquad -\infty<t<+\infty\ ,$$

where $\beta \neq 0$ is a given constant and where h is a given periodic function of period 2π in t and square integrable in the fundamental rectangle $[0,\pi] \times [0,2\pi]$. This is a problem at resonance. Fucik and Mawhin have recently proved that a sufficient condition for existence is that

$$g(-\infty) < (2\pi)^{-2} \int_0^{2\pi} \int_0^{\pi} h(t,s)\, dt\, ds < g(+\infty)\ .$$

In the spirit of our previous work on the subject, we prove this result rather easily.

Report No. 41: L. Cesari and R. Kannan, Solutions of nonlinear wave equations with nonlinear damping.

We consider the problem of existence of periodic solutions of period 2π in t of the following nonlinear wave equation with nonlinear damping:

$$u_{tt} - u_{ss} + \beta(u_t) + g(u) = h(t,s)$$

$$u(t,0) = u(t,\pi) = 0\ ,\ u(t+2\pi,s) = u(t,s)$$

$$0<s<\pi \qquad\qquad -\infty<t<+\infty\ ,$$

where $\beta(u_t)$ is a given function, and where h is a given periodic function of period 2π in t and square integrable in the fundamental rectangle $[0,\pi] \times [0,2\pi]$. Again this is a resonance problem, similar to the one of Report No. 41 but with a nonlinear damping term $\beta(u_t)$. This is a much more difficult situation. The following assumptions are made on the functions β and g:

$$\xi\beta(\xi) \geq \gamma|\xi|^{p+1} \text{ for some } \gamma>0\ ,\ p > 1 \text{ and all } \xi\ ;$$

$$|\beta(\xi)| \leq a+b|\xi|^{p} \text{ for some } a\geq 0,\ b>0, \text{ and all } \xi\ ;$$

β and g are continuous on R, and β is strictly monotone,

$$-C \leq g(\xi) \leq C \text{ for some } C \text{ and all } \xi$$

Then we prove that the problem has at least one weak solution.

Report No. 42: L. Cesari and R. Kannan, An existence theorem for periodic solutions of nonlinear parabolic equations.

We consider here the problem of existence of solutions periodic of period 2π in t of the following equation:

$$u_t - u_{xx} - m^2 u = f(t,x,u) + h(t,x), \quad 0<x<\pi, \quad -\infty<t<+\infty,$$

$$u(t,0) = u(t,\pi) = 0$$

$$u(t+2\pi,x) = u(t,x)$$

where m is an integer. This is a parabolic problem at resonance.

Here we assume that $h(t,x)$ is 2π-periodic in t and square integrable in the fundamental rectangle $G = [0,\pi] \times [0,2\pi]$. Also we assume that $f(t,x,u)$ is continuous, 2π-periodic in t, and satisfying

$$f(t,x,u) \geq \eta_1 > 0 \quad \text{for all} \quad (t,x) \in G, \; u \geq R_o,$$

$$f(t,x,u) \leq -\eta_1 < 0 \quad \text{for all} \quad (t,x) \in G, \; u \leq -R_o;$$

for given $\eta_1, R_o > 0$. We prove that if $|f(t,x,u)| \leq \alpha+\beta|u|^k$ for some constants α,β and $0<k<1$, and if

$$4\pi\eta_1 > \int_0^{2\pi}\int_0^{\pi} h(t,x) \sin mx \, dt \, dx,$$

then the problem has at least a weak solution.

Report No. 45: Patrizia Pucci, Integrali di Riemann e di Burkill-Cesari

In real analysis, Henstock studied a generalization of the Riemann integration process which for instance allows the integration of every function $f(x)$, $a\leq x\leq b$, which is the derivative everywhere of a function $F(x)$ (generalized Riemann integral). Motivated by questions of the calculus of variations and surface area theory, Cesari studied a generalization of the Burkill process of integration of interval functions, namely a process of integration of quasi-additive set functions. This process includes Burkill as well as Weierstrass integrals, and also Perron, Lebesgue, Lebesgue-Stieltjes, and other processes of integration. A great deal of work on this process of integration of quasi-additive set functions has been done recently by Stoddart, Warner, Vinti, Mambriani, Boni, and others. In this Report, Patrizia Pucci shows that the generalized Riemann integral also can be included in Cesari's process of integration of quasi-additive set functions. Moreover, she discusses in detail the process of integration on any L-integrable functions $f(x)$ - thought of as defined almost everywhere only - so as to obtain its absolutely continuous integral function $F(x)$.

Reports Nos. 46, 47, 48: Michael W. Smiley, 1. A weak formulation of the 2-point boundary value problem for hyperbolic equations; 2. Existence theorems for linear hyperbolic boundary value problems at resonance; 3. Existence theorems for nonlinear hyperbolic boundary value problems at resonance.

The author considers hyperbolic systems in a cylinder $D = [0,T] \times G$, G a bounded domain in $\mathbb{R}^n$,

$$\frac{\partial^2 u}{\partial t^2} + Au = \varepsilon g[u], \quad u \in \mathbb{R}^n ,$$

where $u \in \mathbb{R}^n$, A is an elliptic operator in G which may depend on t, and $g : \mathbb{R}^n \to \mathbb{R}^n$ is a given continuous function. The boundary conditions at the ends of the cylinder, that is, at $t = 0$ and $t = T$, are general linear combinations of the unknowns $u(0,x)$, $u(T,x)$ and the derivatives $u'(0,x)$, $u'(T,x)$. In particular, Dirichlet and Neumann boundary conditions are included. Again the boundary conditions on the lateral surface of the cylinder $[0,T] \times \partial G$, are general linear combinations of the unknown, in particular Dirichlet and Neumann conditions are included.

The author frames the problem entirely in the formulation of Lions, that is, the given hyperbolic partial differential system above is treated as an ordinary differential equation on $[0,T]$ whose unknown u has its values in a suitable Banach space of functions on G.

Lions has considered only the Cauchy problem for the linear case. Here the author treats the 2-point boundary problem with nonlinear second members.

Report No. 46 is a preliminary formulation of the weak solutions of the linear problem $\partial^2 u/\partial t^2 + Au = f$. Report No. 47 is a discussion of the 2-point linear problem with existence and uniqueness theorems and relevant estimates. Report No. 48 is a discussion of the nonlinear problem with corresponding existence theorems.

Report No. 49: Shui-Hung Hou, Implicit function theorem in topological spaces.

The author proves here Filippov type implicit function theorems for Carathéodory mappings in general topological spaces. The author also proves a Banach space version which is applicable to optimal control problems. For this purpose, the author introduces the basic concept of separation in topological spaces. Connection between separation properties of spaces and measurability of graphs of multifunctions are established.

Report No. 50: Shui-Hung Hou, On property (Q) and other semicontinuity properties of multifunctions.

Different upper semicontinuity properties of multifunctions in general linear topological spaces are presented and their interrelationships are expounded in detail. In particular criteria are given for Cesari's property (Q) for multifunctions $f : X \to E$ where X is a general topological space and E is a locally convex space. One of the criteria is that f is maximal monotone.

Report No. 51: Shui-Hung Hou, Controllability and feed-back systems.

Using fixed point theorems for set valued functions, existence of admissible pairs is obtained for a feedback control system governed by $Ex(t) = f(t,Mx(t), u(t))$, $u(t) \in w(t,Mx(t))$, $t \in [0,T]$ where $E : X \to Y$ is linear and M is compact, on $X = L_p[[0,T],Y]$ and Y a Banach space. Here w is a measurable multifunction. A special case is considered with $E = R + \Lambda$ where R is maximal monotone and Λ is a linear coercive continuous operator. Connections with controllability are discussed in detail.

Report No. 52: Shui-Hung Hou, Existence theorems of optimal control problems in Banach spaces.

Existence of optimal solutions is proved for Mayer and Lagrange type control problems described by equations of the form $Ex(t) = f(t,Mx(t),u(t))$, $u(t) \in w(t)$, $t \in [0,T]$. Here E is linear maximal monotone and M is compact. The function $f(t,y,u)$ is Carathéodory type and yields operators $N_u x(t) = f(t,Mx(t),u(t))$ which are maximal monotone. The set Φ of admissible trajectories is shown to be weakly relatively sequentially compact in $L_p([0,T]),V)$, $1<p<+\infty$. Evolution type equations are presented as applications.

For the problems of plasma oscillations under an external radiation specifically proposed, the needed equations have been discussed with D. Graffi (Bologna), and P. Bassanini (Roma, in Ann Arbor summer 1980), and they are being investigated. These equations form an hyperbolic system, of which a suitable particularization is reducible to the canonic bicharacteristic form. The papers of Cesari, Kannan, and Smiley, (Reports Nos. 34, 37, 40, 41, 43, 44, 47, 48, 49) cover very simple models of forced oscillations monitered by hyperbolic systems). The specific techniques used by Cesari for the problem of duplication of frequency of laser light, in particular the existence theorems of Cesari, and the methods of successive approximations of Bassanini-Cesari (cf. Report No. 38) seem to yield the desired results.

Summaries prepared by Joel Smoller

During the period in question, I wrote 5 papers, (titles and summaries given below) and I spoke at several conferences in England, France, Germany and Spain.

Papers:

1. On The Vacuum State For The Isentropic Gas Dynamics Equations (with T.P. Liu; Adv. in Appl. Math., Vol. 1, 345-359 (1980)).

   In this paper we show that the vacuum state $(P = 0)$ is the obstruction to the existence of global solutions. We describe some new and novel phenomena which occur near $P = 0$, and we show that the Glimm estimates fail near $P = 0$.

2. Dispersion and Shock-Wave Structure (with R. Shapiro, J. Diff. Equs., to appear)

   We consider the problem of finding shock profiles when dispersion is present in the equations. We show that if the dispersion terms are small with respect to the viscosity terms, then such profiles always exist. In the case of weak shocks, we show that the profiles also exist, in general.

3. Stability and Bifurcation of Steady-State Solutions for the Predator-Prey Equations with Diffusion (with E. Conway and R. Gardner, to appear)

   In this paper we find some new and interesting bifurcation phenomena for a coupled pair of parabolic equations. For example, we show that a steady-state solution can bifurcate into a continuum of such solutions when a parameter crosses a certain value. We also investigate the stability of the steady-state solutions which we construct.

4. Periodic Traveling Waves for Predator-Prey Equations Via the Conley Index (with R. Gardner, to appear).

We construct periodic traveling waves for the predator-prey equations with diffusion. This is equivalent to showing that a certain set of differential equations in $\mathbb{R}^4$ admits a periodic solution. The method is to "perturb" off a singular solution, and use topological methods to show that there are periodic solutions in any small neighborhood of the singular one.

5. Shock Profiles, Entropy and Dispersion (with C. Conley, to appear)

We show that whenever the associated system admits a convex entropy (e.g. as in gas dynamics or magnetohydrodynamics), then shock profiles always exist, even in the presence of strong dispersion terms, provided that the gradient of the nonlinear function is bounded away from zero at ∞ ; i.e. the equations for the profiles are of the form

$$B\dot{u} = v$$
$$\dot{v} = -AB^{-1}v + \nabla \phi (u) \; , \; (u,v) \in \mathbb{R}^n \times \mathbb{R}^n ,$$

and we require $|\nabla \phi (u)| \geq r > o$ if $|u| \geq R$. Here B is the "dispersion" matrix, and A the dissipation matrix.